Exceptional field theories and gauged supergravities
Dualising consistent truncations

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Berman, Blair, EM, Perry arXiv:1303.6727
Blair, EM, Park arXiv:1311.5109
Blair, EM arXiv:1412.0635
EM, Samtleben arXiv:1510.03433
EM, Samtleben to appear

Introduction to EFT

- $E_{d(d)}$ invariant extension of supergravities.
- Similar to generalised geometry $TM \oplus T^*M$, $TM \oplus \Lambda^2 T^*M$, ...
- ...but can describe non-geometric backgrounds thanks to extra coordinates (in representation of $E_{d(d)}$).
- DoFs of 11-dimensional supergravity organise into $E_{d(d)}$ or H_d reps.
- E.g. write $11 = 7 + 4$ to make $E_{7(7)}$ manifest.

$E_{d(d)}$	H_d
$E_{8(8)}$	$SO(16)$
$E_{7(7)}$	$SU(8)$
$E_{6(6)}$	$USp(8)$
$Spin(5,5)$	$Spin(5) \times Spin(5)$
$SL(5)$	$SO(5)$
$SL(2) \times SL(3)$	$SO(2) \times SO(3)$
$SL(2) \times \mathbb{R}^+$	$SO(2)$

Generalised Lie derivative, section condition

- $GL(11) \longrightarrow GL(11-d) \times GL(d) \longrightarrow GL(11-d) \times E_{d(d)}$
- $x^{\hat{\mu}} \quad \longrightarrow \quad (x^\mu, x^i) \quad \longrightarrow \quad (x^\mu, Y^M)$
- $g_{ij},\, C_{ijk},\, C_{ijklmn},\, \ldots \longrightarrow M_{MN}(x,Y) \in E_{d(d)}/H_d$
- $g_{\mu i},\, C_{\mu ij},\, C_{\mu ijklm},\, \ldots \longrightarrow A_\mu{}^M(x,Y)$
- In general, two-forms, etc. appear too. Fermions similar.
- Symmetries also combine into $E_{d(d)}$ action: generalised Lie derivative

$$\mathcal{L}_\xi U^M = \xi^N \partial_N U^M - U^N \partial_N \xi^M + Y^{MP}_{NQ} U^Q \partial_P \xi^N \,, \qquad (1)$$

  Y^{MN}_{PQ} is $E_{d(d)}$ invariant.
- Closure requires "section condition"

$$Y^{MN}_{PQ} \partial_M f \partial_N g = Y^{MN}_{PQ} \partial_M \partial_N f = 0 \,. \qquad (2)$$

Solutions to section condition

- Two independent solutions to section condition.
  (a) $GL(d) \subset E_{d(d)}$ invariant $\Rightarrow$ 11-dimensional supergravity.
  (b) $GL(d-1) \times SL(2) \subset E_{d(d)}$ invariant $\Rightarrow$ IIB supergravity.
- All fields constrained by section condition $Y^{AB}_{CD} \partial_A \otimes \partial_B = 0$ ...
- ...but not truncated! Dependence on (x^μ, Y^A) up to section.
- $\Rightarrow$ we are really describing 11-dimensional SUGRA / IIB SUGRA, and not just on tori
- The space of structures, e.g. $M_{AB}(x, Y)$, are infinite-dimensional $\Rightarrow$ not duality group (yet): $E_{d(d)}/H_d(Y)$

Consistent truncations

- Truncate 11-d / IIB SUGRA on some background keeping only certain modes.
- Yields gauged SUGRA.
- Gauged SUGRAs specified by "embedding tensor" which embeds $G_{gauge} \subset E_{d(d)}$ (for maximal SUSY): $X_M = \theta_M{}^\alpha t_\alpha$.
  - Linear constraint: $\mathbb{P}\theta = 0 \Rightarrow$ only certain irreps allowed.
  - Quadratic constraint: $\mathbb{P}\theta^2 = 0 \Rightarrow$ closure of gauge algebra.
- Consistency $\Rightarrow$ solutions of gauged SUGRA are solutions of full theory.
- Finding consistent truncations normally difficult.

Maximally SUSY truncations

- Maximal SUSY $\Rightarrow$ trivial generalised tangent bundle.
- Internal space is "generalised parallelisable" $\Rightarrow$ well-defined generalised vielbein $U^{\bar{M}}{}_M(Y) \in E_{d(d)}/H(d)$.
- "Generalised Scherk-Schwarz": Truncation Ansatz uses $U^{\bar{M}}{}_M(Y)$ and scalar density $\rho(Y)$.

Generalized Scherk-Schwarz Ansatz

- $M_{MN}(x, Y) = U^{\bar{M}}{}_M(Y) U^{\bar{N}}{}_N(Y)\, \tilde{M}_{\bar{M}\bar{N}}(x)$
- $A_\mu{}^M(x, Y) = \rho^{-1}(Y) U_{\bar{M}}{}^M(Y)\, \tilde{A}_\mu{}^{\bar{M}}(x)$
- etc.

- Moduli space is now $E_{d(d)}/H(d) \Rightarrow$ duality group of maximal gauged SUGRAs.

Maximally SUSY consistent truncation

- Introduce a generalised connection compatible with $U_M{}^{\bar{M}}$.
- Generalised tangent space trivial $\Rightarrow$ no spin-connection.

Weitzenböck connection

Unique connection that preserves $U_M{}^{\bar{M}}$ with vanishing spin-connection

$$\Gamma_{MN}{}^P = U_M{}^{\bar{M}} \partial_N U_{\bar{M}}{}^P \,. \tag{3}$$

- Torsion

$$\mathcal{L}^{\nabla}_{\xi} V^M = \mathcal{L}^{\partial}_{\xi} V^M + \theta_{NP}{}^M \xi^N V^P \,. \tag{4}$$

- For truncation Ansatz $V^M(x,Y) = U^M{}_{\bar{M}}(Y) V^{\bar{M}}(x)$ and $\xi^M(x,Y)$

$$\mathcal{L}_{\xi} V^M(x,Y) = -\theta_{NP}{}^M \xi^N(x,Y) V^P(x,Y) \,. \tag{5}$$

Maximally SUSY consistent truncation

- In EFT, $\theta_{\bar{N}\bar{P}}{}^{\bar{M}}$ appears like embedding tensor of maximal gSUGRA.
- $\theta_{\bar{M}\bar{N}}{}^{\bar{P}}$ satisfies LC of maximal gSUGRA.
- Section condition $\Rightarrow \theta_{\bar{M}\bar{N}}{}^{\bar{P}}$ satisfies QC (*not necessary*).
- Constant $\theta_{\bar{M}\bar{N}}{}^{\bar{P}} \Rightarrow$ *consistent* truncation to maximal gSUGRA.
- Finding consistent truncation $\Rightarrow$ finding the right parallelisation.
- EFT $\Rightarrow$ efficient way to "uplift" gSUGRAs to IIB / 11-dimensional SUGRA.
- Important to develop tools to construct uplifts.

"Dualising" consistent truncations: 7-d

- 7-d max gSUGRAs, $E_{4(4)} = SL(5)$ global symmetry group.
- Linear constraint:

$$\theta = S_{\bar{a}\bar{b}} \oplus Z^{\bar{a}\bar{b},\bar{c}} \oplus \tau_{\bar{a}\bar{b}} \qquad (6)$$

$$\mathbf{15} \oplus \overline{\mathbf{40}} \oplus \mathbf{10}$$

- $CSO(p,q,r)$ gaugings from M-theory / IIA on S^3 and $H^{p,q}$. Hohm, Samtleben 1410.8145
- $CSO(p,q,r)$ from IIB on $H^{p,q}$?
- Can these be related?
- Is there a "duality" relating IIA and IIB consistent truncations?
- When do consistent IIA truncations imply IIB truncations?
- EFT useful tool – even when everything geometric and "on section".

7-d EFT basics

- 4 spacetime coordinates + 6 wrapping coordinates $\Rightarrow$ 10-dimensional extended space: $Y^{[ab]}$. $a, b = 1, \ldots, 5$
- Section condition $\partial_{[bc} A \, \partial_{de]} B = 0$.
- Two inequivalent solutions to section condition:
  - M-theory (IIA); only depend on 4 coords $Y^{\alpha 5}$, $\alpha = 1, \ldots, 4$.
  - IIB; only depend on 3 coords $Y^{\mu\nu}$, $\mu, \nu = 1, \ldots 3$.
- Can we relate the IIA / IIB solutions of section condition?
- Something like "T-duality". But NOT strictly a duality.

“T-duality” in EFT

- Decompose $SL(5) \longrightarrow SL(4) \sim Spin(3,3)$.
- $\bar{a} = 1, \ldots, 5 = (\bar{\alpha}, \bar{5})$ with $\bar{\alpha} = 1, \ldots 4$ $SL(4)$ indices.

$$\begin{aligned} S_{\bar{a}\bar{b}} &\longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}} \,, \\ \mathbf{15} &\longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1} \,. \end{aligned} \tag{7}$$

$$\begin{aligned} Z^{\bar{a}\bar{b},\bar{c}} &\longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}} \,, \\ \overline{\mathbf{40}} &\longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}} \,. \end{aligned} \tag{8}$$

$$\begin{aligned} \mathcal{T}_{\bar{a}\bar{b}} &\longrightarrow \mathcal{T}_{\bar{\alpha}\bar{\beta}} \oplus \mathcal{T}_{\bar{\alpha}\bar{5}} \,, \\ \mathbf{10} &\longrightarrow \mathbf{6} \oplus \mathbf{4} \,. \end{aligned} \tag{9}$$

- How can we exchange the **10**’s, **6**’s and **4**’s?

"T-duality" in EFT

- Decompose $SL(5) \longrightarrow SL(4) \sim Spin(3,3)$.
- $\bar{a} = 1, \ldots, 5 = (\bar{\alpha}, \bar{5})$ with $\bar{\alpha} = 1, \ldots 4$ $SL(4)$ indices.

IIA $CSO(p,q,r)$

$$\begin{aligned} S_{\bar{a}\bar{b}} &\longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}} \,, \\ \mathbf{15} &\longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1} \,. \end{aligned} \tag{7}$$

IIB $CSO(p,q,r)$

$$\begin{aligned} Z^{\bar{a}\bar{b},\bar{c}} &\longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}} \,, \\ \overline{\mathbf{40}} &\longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}} \,. \end{aligned} \tag{8}$$

$$\begin{aligned} \tau_{\bar{a}\bar{b}} &\longrightarrow \tau_{\bar{\alpha}\bar{\beta}} \oplus \tau_{\bar{\alpha}\bar{5}} \,, \\ \mathbf{10} &\longrightarrow \mathbf{6} \oplus \mathbf{4} \,. \end{aligned} \tag{9}$$

- How can we exchange the **10**'s, **6**'s and **4**'s?

"T-duality" in EFT: outer automorphism

- Consider IIA / IIB by dimensional reduction $\partial_{\alpha 5} = 0$, i.e. only 6 coordinates $Y^{\alpha\beta}$. No section condition yet.

Ansatz

$$U_a{}^{\bar{a}} = \begin{pmatrix} \omega^{-1/2} V_\alpha{}^{\bar{\alpha}} & 0 \\ 0 & \omega^2 \end{pmatrix}, \qquad |V| = 1\,. \tag{10}$$

- Define $\partial^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\partial_{\gamma\delta}$. S.c. $\Rightarrow \partial^{\alpha\beta}\partial_{\alpha\beta} = 0$.
- Only non-zero gaugings are **10**'s and **6**'s:

$$\begin{aligned}
S_{\bar{\alpha}\bar{\beta}} &= 4\rho^{-1}\omega\left(V_{(\bar{\alpha}}{}^{\alpha}\partial_{|\alpha\beta|}V_{\bar{\beta})}{}^{\beta}\right), \qquad Z^{\bar{5}(\bar{\alpha},\bar{\beta})} = \rho^{-1}\omega\left(V_\alpha{}^{(\bar{\alpha}}\partial^{|\alpha\beta|}V_\beta{}^{\bar{\beta})}\right), \\
2\tau_{\bar{\alpha}\bar{\beta}} &= -\rho^{-1}\omega\left(\partial_{\alpha\beta}V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} - 5V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta}\partial_{\alpha\beta}\ln\omega + 6V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta}\partial_{\alpha\beta}\ln\left(\rho^{-1}\omega\right)\right), \\
6Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} &= \rho^{-1}\omega\left(\partial^{\alpha\beta}V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}} - 5V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}}\ln\omega\right).
\end{aligned} \tag{11}$$

Consistency condition: gaugings must be constant.

"T-duality" in EFT: outer automorphism

$$S_{\bar{\alpha}\bar{\beta}} = 4\rho^{-1}\omega\left(V_{(\bar{\alpha}}{}^{\alpha}\partial_{|\alpha\beta|}V_{\bar{\beta})}{}^{\beta}\right)\,, \qquad Z^{\bar{5}(\bar{\alpha},\bar{\beta})} = \rho^{-1}\omega\left(V_{\alpha}{}^{(\bar{\alpha}}\partial^{|\alpha\beta|}V_{\beta}{}^{\bar{\beta})}\right)\,,$$

$$2\tau_{\bar{\alpha}\bar{\beta}} = -\rho^{-1}\omega\left(\partial_{\alpha\beta}V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} - 5V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta}\partial_{\alpha\beta}\ln\omega + 6V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta}\partial_{\alpha\beta}\ln\left(\rho^{-1}\omega\right)\right)\,,$$

$$6Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} = \rho^{-1}\omega\left(\partial^{\alpha\beta}V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}} - 5V_{\alpha\beta}{}^{\bar{\alpha}\bar{\beta}}\ln\omega\right)\,.$$

"Duality" transformation: outer automorphism of $SL(4)$

$$V_{\alpha}{}^{\bar{\alpha}} \longleftrightarrow \left(V^{-T}\right)_{\bar{\alpha}}{}^{\alpha}\,, \qquad \partial_{\alpha\beta} \longleftrightarrow \partial^{\alpha\beta}\,. \tag{12}$$

$$S_{\bar{\alpha}\bar{\beta}} \longleftrightarrow Z^{\bar{5}(\bar{\alpha},\bar{\beta})}\,, \qquad \tau_{\bar{\alpha}\bar{\beta}} \longleftrightarrow \frac{1}{2}\epsilon^{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}}\tau_{\bar{\gamma}\bar{\delta}}\,, \qquad Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \longleftrightarrow \frac{1}{2}\epsilon_{\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\delta}}Z^{\bar{5}[\bar{\gamma},\bar{\delta}]}$$

- Swaps solutions of section condition.
- NS-NS fields remain invariant!
- Given //-isation of **10**'s and **6**'s, we get //-isation of "dual gaugings".
- Example: Spheres and hyperboloids of IIB.

A no-go theorem

$$S_{\bar{a}\bar{b}} \longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}}\,, \qquad (13)$$
$$\mathbf{15} \longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1}\,.$$

IIA $CSO(p,q,r)$ → $\mathbf{10}$

$$Z^{\bar{a}\bar{b},\bar{c}} \longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}}\,, \qquad (14)$$
$$\overline{\mathbf{40}} \longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}}\,.$$

IIB $CSO(p,q,r)$ → $\overline{\mathbf{10}}$

$$\tau_{\bar{a}\bar{b}} \longrightarrow \tau_{\bar{\alpha}\bar{\beta}} \oplus \tau_{\bar{\alpha}\bar{5}}\,, \qquad (15)$$
$$\mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}\,.$$

A no-go theorem

$$S_{\bar{a}\bar{b}} \longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}}\,,$$
$$\mathbf{15} \longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1}\,. \tag{13}$$

IIB?? $CSO(p,q,r)$ → $\mathbf{10}$

$$Z^{\bar{a}\bar{b},\bar{c}} \longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}}\,,$$
$$\overline{\mathbf{40}} \longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}}\,. \tag{14}$$

IIA?? $CSO(p,q,r)$ → $\overline{\mathbf{10}}$

$$\tau_{\bar{a}\bar{b}} \longrightarrow \tau_{\bar{\alpha}\bar{\beta}} \oplus \tau_{\bar{\alpha}\bar{5}}\,,$$
$$\mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}\,. \tag{15}$$

A no-go theorem

- Assume we have IIA gauging, i.e. only $Y^{\mu 4}$ dependence.
- Consistency conditions then imply

Necessary requirement for IIA gauging

$$\left(-Z^{\bar{a}\bar{b},\bar{c}} + 3\epsilon^{\bar{a}\bar{b}\bar{c}\bar{d}\bar{e}}\tau_{\bar{d}\bar{e}}\right) U_{\bar{a}\bar{b}}{}^{54} = \left(-Z^{\bar{a}\bar{b},\bar{c}} + 3\epsilon^{\bar{a}\bar{b}\bar{c}\bar{d}\bar{e}}\tau_{\bar{d}\bar{e}}\right) U_{\bar{a}\bar{b}}{}^{4\mu} = 0\,. \qquad (16)$$

- Try and gauge $\overline{\mathbf{10}}$, i.e. $Z^{\bar{5}(\bar{\alpha},\bar{\beta})} = \eta^{\bar{\alpha}\bar{\beta}}$, from IIA.
- We find twist matrices must vanish.
- $\overline{\mathbf{10}}$ only comes from IIB and by "duality" $\mathbf{10}$ only comes from IIA.

Dualising the **4**'s

$$S_{\bar{a}\bar{b}} \longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}}\,,$$

$$\mathbf{15} \longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1}\,.$$

$$Z^{\bar{a}\bar{b},\bar{c}} \longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}}\,,$$

$$\overline{\mathbf{40}} \longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}}\,.$$

$$\mathcal{T}_{\bar{a}\bar{b}} \longrightarrow \mathcal{T}_{\bar{\alpha}\bar{\beta}} \oplus \mathcal{T}_{\bar{\alpha}\bar{5}}\,,$$

$$\mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}\,.$$

Dualising the **4**'s

$$S_{\bar{a}\bar{b}} \longrightarrow S_{\bar{\alpha}\bar{\beta}} \oplus S_{\bar{\alpha}\bar{5}} \oplus S_{\bar{5}\bar{5}}\,,$$

$$\mathbf{15} \longrightarrow \mathbf{10} \oplus \mathbf{4} \oplus \mathbf{1}\,.$$

$$Z^{\bar{a}\bar{b},\bar{c}} \longrightarrow Z^{\bar{\alpha}\bar{\beta},\bar{\gamma}} \oplus Z^{\bar{5}(\bar{\alpha},\bar{\beta})} \oplus Z^{\bar{5}[\bar{\alpha},\bar{\beta}]} \oplus Z^{\bar{5}\bar{\alpha},\bar{5}}\,,$$

$$\overline{\mathbf{40}} \longrightarrow \overline{\mathbf{20}} \oplus \overline{\mathbf{10}} \oplus \mathbf{6} \oplus \overline{\mathbf{4}}\,.$$

$$\tau_{\bar{a}\bar{b}} \longrightarrow \tau_{\bar{\alpha}\bar{\beta}} \oplus \tau_{\bar{\alpha}\bar{5}}\,,$$

$$\mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}\,.$$

Can we dualise the **4**'s?

Dualising the $\mathbf{4}$'s

- Previous Ansatz $\Rightarrow$ only gaugings $\in$ $\mathbf{10}$'s $\oplus$ $\mathbf{6}$'s.
- Relax twist Ansatz $\Rightarrow$ gaugings of the $\mathbf{4}$'s.
- In general, we find the gaugings cannot be dualised.
- Gaugings of $\mathbf{4} \subset \mathbf{10}, \mathbf{15}$ more constrained than $\overline{\mathbf{4}} \subset \overline{\mathbf{40}}$.
- The two $\mathbf{4}$'s are not on an equal footing.

$$\begin{aligned} \mathbf{4} \subset \mathbf{15} &\longrightarrow \overline{\mathbf{4}} \subset \overline{\mathbf{40}}\,, \\ \overline{\mathbf{4}} \subset \overline{\mathbf{40}} &\not\longrightarrow \mathbf{4} \subset \mathbf{15}\,. \end{aligned} \tag{17}$$

- The dual $\mathbf{4}$ gaugings in general violate the quadratic constraint!

Summary of dualising 7-d gaugings

Theorem 1

Given a maximally SUSY consistent truncation of IIA / IIB with gaugings only in $\mathbf{10} \oplus \mathbf{6} \in SL(4)$, there is a "dual" maximal SUSY consistent truncation of IIB / IIA on same NS background yielding an inequivalent maximal gauged SUGRA.

Theorem 2

$CSO(p, q, r)$ gaugings in $\mathbf{10} \subset \mathbf{15}$ cannot come from IIB, those in $\mathbf{10}' \subset \mathbf{40}'$ cannot come from IIA.

- Theorem 1 does not hold in general for gaugings of the $\mathbf{4}$'s. Counterexamples!
- Gaugings of $\mathbf{4}$'s must be analysed example-by-example.
- How about in other dimensions?

Dualising 4-d gaugings

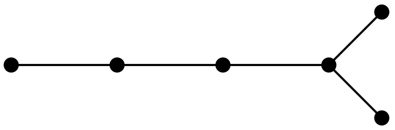
- 4-d maximal gSUGRA $\Rightarrow E_{7(7)}$ global symmetry group.
- Extended coordinates: $Y^A \in \mathbf{56}$.
- Linear constraint: $\Theta \in \mathbf{912} \oplus \mathbf{56}$.
- T-duality subgroup: $SO(6,6) \times SL(2) \subset E_{7(7)}$.
- Break $E_{7(7)} \to SO(6,6) \times SL(2)$:

$$\begin{aligned} \mathbf{56} &\to (\mathbf{12},\mathbf{2}) \oplus (\mathbf{32},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \,, \\ \mathbf{912} &\to (\mathbf{220},\mathbf{2}) \oplus \left(\mathbf{352}',\mathbf{1}\right) \oplus (\mathbf{32},\mathbf{1}) \oplus (\mathbf{12},\mathbf{2}) \,. \end{aligned} \tag{18}$$

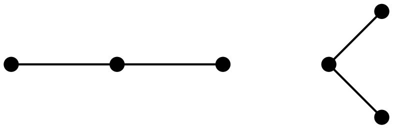
- How do we best understand the outer automorphism of $SO(6,6)$ here?
- Can we uplift the $SO(4) \times SO(2,2) \ltimes T^{16}$ gSUGRAs with Minkowski vacua? Dall'Agata, Inverso 1112.3345; Baguet, Pope, Samtleben 1510.08926
- Uplifting more maximal gauged SUGRAs.

Dualising 4-d gaugings

- Consider $SO(6,6) \to SL(4) \times SL(4)$
- In terms of Dynkin diagrams we have



breaking up as



- 3 outer automorphisms: 1 per $SL(4)$ + interchange of $SL(4)$'s.
- Up to field redefinitions, only 1 $SL(4)$ outer automorphism is physical.
- $\mathbf{32} \to (\mathbf{4}, \mathbf{4}) \oplus (\mathbf{4}', \mathbf{4}')$.
- $\mathbf{32}' \to (\mathbf{4}, \mathbf{4}') \oplus (\mathbf{4}', \mathbf{4})$.

$E_{7(7)}$ section condition revisited

- Recall $E_{7(7)} \to SO(6,6) \times \mathrm{SL}(2)$, $\mathbf{56} \to (\mathbf{2},\mathbf{12}) \oplus (\mathbf{1},\mathbf{32})$
- Section condition ($M = 1, \ldots, 56$)

$$\Omega^{MN}\partial_M\partial_N = 0\,, \qquad (t_\alpha)^{MN}\,\partial_M\partial_N = 0 \tag{19}$$

- $A = 1, \ldots, 12$, $i = 1, 2$, $\mathcal{A} = 1, \ldots, 32$, $\dot{\mathcal{A}} = \dot{1}, \ldots, \dot{32}$
- Implies $\partial_{A,2} = 0$. Define $\partial_{A,1} = \partial_A$.
- We have

$$\eta^{AB}\partial_A\partial_B = 0\,, \qquad \left(\Gamma^C\right)_{\dot{\mathcal{A}}}{}^{\mathcal{B}}\partial_C\partial_{\mathcal{B}} = 0\,, \qquad \left(\Gamma^{AB}\right)^{\mathcal{A}\mathcal{B}}\partial_{\mathcal{A}}\partial_{\mathcal{B}} = 0\,. \tag{20}$$

$E_{7(7)}$ section condition revisited

- Break $SO(6,6) \to SL(4) \times SL(4)$.
- $\mathbf{12} \to (\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{6})$
- $\mathbf{32} \to (\mathbf{4},\mathbf{4}) \oplus (\mathbf{4}',\mathbf{4}')$
- Use $\alpha = 1,\ldots,4$, $\dot\alpha = 1,\ldots,4$, $\partial^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\partial_{\gamma\delta}$, $\partial^{\dot\alpha\dot\beta}$

$$\eta^{AB}\partial_A\partial_B = 0 \quad \Rightarrow \quad \partial^{\alpha\beta}\partial_{\alpha\beta} = 0\,, \qquad \partial^{\dot\alpha\dot\beta}\partial_{\dot\alpha\dot\beta} = 0\,. \tag{21}$$

- $\Gamma\partial\partial = 0$ constrain derivatives $\partial_{\alpha\dot\alpha}$ and $\partial^{\alpha\dot\alpha}$.
- Two inequivalent solutions to (21):
  (a) $\partial_{\mu\nu} = 0\,, \qquad \partial_{\dot\mu\dot\nu} = 0 \quad \Rightarrow \text{only } Y^{\mu 4},\ Y^{\dot\mu\dot 4}\,, \qquad \mu = 1,\ldots,3\,.$
  (b) $\partial_{\mu 4} = 0\,, \qquad \partial_{\dot\mu\dot\nu} = 0 \quad \Rightarrow \text{only } Y^{\mu\nu},\ Y^{\dot\mu\dot 4}\,, \qquad \dot\mu = \dot 1,\ldots,\dot 3\,.$
- These give 6 coordinates: one extendable to 7-d, other not extendable.
- (a) symmetric under $SL(4) \leftrightarrow SL(4)$, can be extended by one coordinate $\partial_{4\dot 4} \neq 0 \Rightarrow$ IIA / 11-d
- (b) symmetric under $SL(4) \leftrightarrow SL(4)$ + outer automorphisms. There is no single coordinate invariant under this $\Rightarrow$ inextendable $\Rightarrow$ IIB

Section choices and "duality"

- We saw two independent solutions to section

  (a) $\partial_{\mu\nu} = 0\,, \qquad \partial_{\dot{\mu}\dot{\nu}} = 0\,, \qquad \mu = 1, \ldots, 3\,.$
  (b) $\partial_{\mu 4} = 0\,, \qquad \partial_{\dot{\mu}\dot{\nu}} = 0\,, \qquad \dot{\mu} = \dot{1}, \ldots, \dot{3}\,.$

- (a) is IIA, (b) is IIB.
- $SL(4)$ outer automorphism $\partial_{\alpha\beta} \leftrightarrow \partial^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} \Rightarrow$ IIA $\leftrightarrow$ IIB.

Decomposing twist matrices under $SO(6,6)$

- $E_{7(7)} \to SO(6,6) \times SL(2)$.
- $\mathbf{133} \to (\mathbf{66}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus \left(\mathbf{32}', \mathbf{2}\right)$.
- $\mathbf{912} \to (\mathbf{220}, \mathbf{2}) \oplus \left(\mathbf{352}', \mathbf{1}\right) \oplus (\mathbf{32}, \mathbf{1}) \oplus (\mathbf{12}, \mathbf{2})$.

$SO(6,6)$ Ansatz

$$U_M{}^{\bar{M}} = \exp\left(\frac{1}{2}\omega_{AB} t^{[AB]} + \frac{1}{2}\kappa_{ij} t^{(ij)}\right) , \qquad \kappa_{ij} = \begin{pmatrix} \kappa & 0 \\ 0 & -\kappa \end{pmatrix} . \qquad (22)$$

- With this Ansatz we get only one of $SL(2)$ multiplets $(\mathbf{220}, \mathbf{2}) \oplus 2 \times (\mathbf{12}, \mathbf{2})$.

Decomposing twist matrices under $SL(4)$

- Discuss outer automorphism $\Rightarrow SO(6,6) \to SL(4) \times SL(4)$.
- $\mathbf{220} \to (\mathbf{1},\mathbf{10}) \oplus (\mathbf{1},\mathbf{10'}) \oplus (\mathbf{10},\mathbf{1}) \oplus (\mathbf{10'},\mathbf{1}) \oplus (\mathbf{6},\mathbf{15}) \oplus (\mathbf{15},\mathbf{6})$.
- Recall $\mathbf{12} \to (\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{6})$.

$SL(4) \times SL(4)$ Ansatz

$$U_A{}^{\bar{A}} \to V_{[\alpha}{}^{[\bar{\alpha}} V_{\beta]}{}^{\bar{\beta}]} \oplus W_{[\dot{\alpha}}{}^{[\bar{\dot{\alpha}}} W_{\dot{\beta}]}{}^{\bar{\dot{\beta}}]} ,$$
$$V_\alpha{}^{\bar{\alpha}}(Y^{\gamma\delta}) , \quad W_{\dot{\alpha}}{}^{\bar{\dot{\alpha}}}(Y^{\dot{\gamma}\dot{\delta}}) , \quad |V| = |W| = 1 . \qquad (23)$$

- This only excites the $\mathbf{10}$, $\mathbf{10'}$ and $\mathbf{6}$'s in the above decomposition.

$$S_{\bar{\alpha}\bar{\beta}} \propto V_{(\bar{\alpha}}{}^{\alpha} \partial_{|\alpha\beta|} V_{\bar{\beta})}{}^{\beta} , \qquad S_{\bar{\dot{\alpha}}\bar{\dot{\beta}}} \propto W_{(\bar{\dot{\alpha}}}{}^{\dot{\alpha}} \partial_{|\dot{\alpha}\dot{\beta}|} W_{\bar{\dot{\beta}})}{}^{\dot{\beta}} ,$$
$$Z^{\bar{\alpha}\bar{\beta}} \propto V_\alpha^{(\bar{\alpha}} \partial^{\alpha\beta} V_\beta{}^{\bar{\beta})} , \qquad Z^{\bar{\dot{\alpha}}\bar{\dot{\beta}}} \propto W_{\dot{\alpha}}{}^{(\bar{\dot{\alpha}}} \partial^{\dot{\alpha}\dot{\beta}} W_{\dot{\beta}}{}^{\bar{\dot{\beta}})} , \qquad (24)$$
$$\tau_{\bar{\alpha}\bar{\beta}} \sim \partial_{\alpha\beta} V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} + V_{\bar{\alpha}\bar{\beta}}{}^{\alpha\beta} \partial_{\alpha\beta} \ln \rho\kappa$$

Uplift and duality

- Uplift gaugings which only excite the **10**'s and **6**'s.
- Includes truncation of IIA / IIB on 6-d $H^{p,q} \times H^{r,s}$ ...
- ... including uplift $SO(4) \times SO(2,2)$ gSUGRA with Minkowski vacuum.
- Our duality gives another IIB / IIA truncation to $SO(4) \times SO(2,2)$ gSUGRA.
- Are these gSUGRAs equivalent??

Another look at 7-d inequivalence

- Recall "duality" gives inequivalent gaugings in 7-d.
- Obvious because it relates different irreps.
- Obvious because gauge groups are different.
- Gaugings couple to vector fields as ${A_\mu}^M X_M$, $X_M = {\theta_M}^\alpha t_\alpha$.
- Here $X_M \to X_{ab}$, ${A_\mu}^M \to {A_\mu}^{ab}$ so

$$ {A_\mu}^{ab} X_{ab} = {A_\mu}^{ab} {\theta_{ab,c}}^d {t_d}^c \,. \tag{25} $$

- Consider again $SL(5) \to SL(4)$, embedding tensor f_{ABC} in $\mathbf{10} \oplus \overline{\mathbf{10}}$:

$$ \begin{aligned} \mathbf{10} &\to \mathbf{6} \oplus \mathbf{4}\,, \quad {A_\mu}^{ab} \to {A_\mu}^{\alpha\beta} \oplus {A_\mu}^{\alpha}\,, \\ \mathbf{24} &\to \mathbf{15} \oplus \mathbf{4} \oplus \overline{\mathbf{4}} \oplus \mathbf{1}\,, \quad {t_a}^b \to {t_\alpha}^\beta \oplus t_\alpha \oplus t^\alpha \oplus t\,, \end{aligned} \tag{26} $$

$$ {A_\mu}^{ab} {\theta_{ab,c}}^d {t_d}^c = {A_\mu}^{\alpha\beta} S_{\alpha\gamma} {t_\beta}^\gamma + A_{\mu\,\alpha\beta} Z^{\alpha\gamma} {t_\gamma}^\beta + {A_\mu}^\alpha S_{\alpha\beta} t^\beta \,. $$

- Final term enhances 1/2-max to max SUSY
- Breaks symmetry between dual gaugings: 10 vs 6 vectors.

(In)equivalence of 4-d gaugings?

- Consider now for $E_{7(7)} \to SO(6,6) \times SL(2)$
- $\mathbf{56} \to (\mathbf{12}, \mathbf{2}) \oplus (\mathbf{32}, \mathbf{1})$
- $\mathbf{133} \to (\mathbf{66}, \mathbf{1}) \oplus (\mathbf{32}', \mathbf{2}) \oplus (\mathbf{1}, \mathbf{3})$
- Embedding tensor only in $(\mathbf{220}, \mathbf{2})$, coupling:

$$A_\mu{}^M \theta_M{}^\alpha t_\alpha = A_\mu{}^{A\,i} f_{ABC\,i} t^{BC} + A_\mu{}^{\mathcal{A}} f_{ABC\,i} \Gamma^{ABC}{}_{\mathcal{A}\dot{\mathcal{A}}} t^{\dot{\mathcal{A}}\,i} \,. \tag{27}$$

- Expect second term to break symmetry between "dual" gaugings.
- $SO(6,6) \to SL(4) \times SL(4)$
- $\mathbf{12} \to (\mathbf{6}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6})$, $A_\mu{}^A \to A_\mu^{\alpha\beta} \oplus A_\mu^{\dot\alpha\dot\beta}$.
- $\mathbf{32} \to (\mathbf{4}, \mathbf{4}) \oplus (\mathbf{4}', \mathbf{4}')$, $A_\mu{}^{\mathcal{A}} \to A_\mu{}^{\alpha\dot\alpha} \oplus A_{\mu\,\alpha\dot\alpha}$.
- $\mathbf{66} \to (\mathbf{1}, \mathbf{15}) \oplus (\mathbf{15}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{6})$, $t^A \to t_\alpha{}^\beta \oplus t_{\dot\alpha}{}^{\dot\beta} \oplus t_{\alpha\beta,\dot\alpha\dot\beta}$.
- $\mathbf{32}' \to (\mathbf{4}, \mathbf{4}') \oplus (\mathbf{4}', \mathbf{4})$, $t^{\dot{\mathcal{A}}} \to t_\alpha{}^{\dot\alpha} \oplus t_{\dot\alpha}{}^\alpha$.

(In)equivalence of 4-d gaugings?

- Consider **220** embedding tensor with only **10**'s excited.
- Coupling is

$$
\begin{aligned}
A_\mu{}^M \theta_M{}^\alpha t_\alpha = {} & A_\mu{}^{\alpha\beta} S_{\beta\gamma} t_\alpha{}^\gamma + A_{\mu\,\alpha\beta} Z^{\beta\gamma} t_\gamma{}^\alpha \\
& + A_\mu{}^{\dot\alpha\dot\beta} S_{\dot\beta\dot\gamma} t_{\dot\alpha}{}^{\dot\gamma} + A_{\mu\,\dot\alpha\dot\beta} Z^{\dot\beta\dot\gamma} t_{\dot\gamma}{}^{\dot\alpha} \\
& + A_\mu{}^{\alpha\dot\alpha} S_{\alpha\beta} t_{\dot\alpha}{}^\beta + A_{\mu\,\alpha\dot\alpha} Z^{\alpha\beta} t_\beta{}^{\dot\alpha} \\
& + A_\mu{}^{\alpha\dot\alpha} S_{\dot\alpha\dot\beta} t_\alpha{}^{\dot\beta} + A_{\mu\,\alpha\dot\alpha} Z^{\dot\alpha\dot\beta} t_{\dot\beta}{}^\alpha \,.
\end{aligned}
\tag{28}
$$

- Coupling symmetric wrt $SL(4)$ outer automorphism $S_{\alpha\beta} \leftrightarrow Z^{\alpha\beta} \Rightarrow$ same gauge group!
- Both "dual" gaugings couple to 28 vectors.
- Is there a more refined way to determine (in)equivalence?

Conclusions and further work

- EFT useful tool to find consistent truncations even for geometric cases!
- "Duality" between IIA/IIB truncations gives inequivalent 7-d gSUGRAs with gaugings in **10** and **6**.
- New consistent truncation of IIB on $H^{p,q}$ to 7-d.
- No-go theorem for 7-d gaugings.
- "Duality" between IIA/IIB truncations to 4-d.
- New consistent truncations of IIA/IIB on $H^{p,q} \times H^{r,s}$ to 4-d.
- Uplift of $SO(4) \times SO(2,2)$ gSUGRA with Minkowski vacuum.

- Does the "duality" give equivalent 4-d gSUGRAs?
- More general uplifts?
- Less SUSY-truncations, e.g. $\mathcal{N} = 2$ gSUGRA?