1/2-maximal consistent truncations using EFT and the M-theory / heterotic duality

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Outline

- Motivation
- Review of maximal truncations
- G_{half}-structures and 1/2-maximal SUSY backgrounds in EFT
- Rewriting EFT with G_{half} manifest
- Truncation Ansatz
- Consistency conditions and embedding tensor
- From EFT to Heterotic DFT

Motivation

- Very successful application of EFT/DFT/EGG.
- Uplifting gauged SUGRAs as consistent truncation of 11-d and type II SUGRA (c.f. Samtleben's talk).
- Geometric understanding of embedding tensor in terms of exceptional generalised geometry.
- So far: max SUSY-preserving truncations, e.g.
  - lower-dimensional max gSUGRA from 11-d or $\mathcal{N} = 2$ 10-d.
  - lower-dimensional 1/2-max gSUGRA from $\mathcal{N} = 1$ 10-d.

Motivation

- What about more general 1/2-max truncations?
  - e.g. 11-d on K3 $\rightarrow$ D$=$7 1/2-max SUGRA with 19 vector multiplets
  - e.g. D$=$7 1/2-max gSUGRA with θ (de Sitter vacuum)
- How do we relate EFT and heterotic SUGRA? (e.g. M-theory / heterotic duality)
- Caveat: I will only address global questions within EGG.
- Note: Whenever referring to SUSY, I mean of the gSUGRA, not of a potential vacuum.
  $\Rightarrow$ no *Killing* spinors required.

Review of maximal consistent truncations

- Consider a truncation $M_{11} = M_D \times M_{11-D}$ (twisted).
- Coordinates (x, y)
- In principle truncation breaks SUSY because of M_{11-D}.
- M_{11-D} admits N spinors $\Theta(y) \Rightarrow$ determines number of SUSY's of D-dimensional theory.
- Max SUSY $\Leftrightarrow$ full set of globally-defined (generalised) spinors
  $\Leftrightarrow$ generalised tangent bundle is trivial
  $\Leftrightarrow$ background has generalised identity structure

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Identity structures

- Trivial generalised tangent bundle
  $\Leftrightarrow$ Id-structure $\Leftrightarrow$ globally well-defined $E^{\bar{M}}{}_M$, κ (scalar density).
- $E^{\bar{M}}{}_M$, κ can be expressed in terms of the spinors.
- Natural connection: Id-connection, i.e.

$$\nabla^{Id}_M E^{\bar{M}}{}_N = \partial_M E^{\bar{M}}{}_N - \Gamma^{Id}_{MN}{}^P E^{\bar{M}}{}_P = 0 = \nabla^{Id}_M \kappa \,.$$

- Preserves each $E^{\bar{M}}{}_M$ **individually** (i.e. without spin-connection).
- Action can be rewritten using (intrinsic) torsion of Id-connection.

Maximal truncation Ansatz

- Truncation Ansatz: "generalised Scherk-Schwarz".
- Factorise Id-structure in terms of background (Y) $\times$ fluctuations (x):

$$E^{\bar{M}}{}_{M}(x,Y) = \mathcal{V}^{\bar{M}}{}_{\hat{M}}(x)\, U^{\hat{M}}{}_{M}(Y)\,, \qquad \kappa(x,Y) = |e|(x)\,\rho(Y)\,.$$

- Twist $U^{\hat{M}}{}_{M}(Y)$ is background Id-structure (vielbein) $\Rightarrow U^{\hat{M}}{}_{M}(Y) \in E_{d(d)}$.
- $U^{\hat{M}}{}_{M}(Y)$ maps $E_{d(d)}$ structure group to $E_{d(d)}$ global symmetry (U-duality group) of maximal gauged SUGRA.
- Truncation Ansatz of other fields determined by their representation.

$$A_{\mu}{}^{M}(x,Y) = \mathcal{A}_{\mu}{}^{\hat{M}}(x)\,\rho(Y)\,U^{M}{}_{\hat{M}}(Y)\,, \ldots\,.$$

Consistency of maximal truncation Ansatz

- Embedding tensor $X_{\hat{M}\hat{N}}{}^{\hat{P}}(U)$ is intrinsic torsion of background:

$$X_{\hat{M}\hat{N}}{}^{\hat{P}} = \mathcal{L}_{U_{\hat{M}}} U_{\hat{N}}{}^{M} U^{\hat{P}}{}_{M} \,.$$

- X satisfies linear constraint automatically, section condition $\Rightarrow$ quadratic constraint.
- All dependence on Y in action appears through $X(U)$ only.
- X constant $\Rightarrow$ consistent truncation Ansatz.

Backgrounds admitting 16 SUSY's in EFT

- Want 1/2-max gSUGRA ⇒ background admitting spinors corresponding to 16 supercharges.
- These backgrounds have generalised $\mathrm{G}_{\mathrm{half}}$ structure (c.f. Strickland-Constable's talk).

D	$E_{d(d)}$	H_d	$\mathrm{G}_{\mathrm{half}}$	G_R
7	$\mathrm{SL}(5)$	$\mathrm{USp}(4)$	$\mathrm{SU}(2)$	$\mathrm{SU}(2)$
6a	$\mathrm{Spin}(5,5)$	$\mathrm{USp}(4)\times\mathrm{USp}(4)$	$\mathrm{SU}(2)\times\mathrm{SU}(2)$	$\mathrm{SU}(2)\times\mathrm{SU}(2)$
6b	$\mathrm{Spin}(5,5)$	$\mathrm{USp}(4)\times\mathrm{USp}(4)$	$\mathrm{USp}(4)$	$\mathrm{USp}(4)$
5	$E_{6(6)}$	$\mathrm{USp}(8)$	$\mathrm{USp}(4)$	$\mathrm{USp}(4)$
4	$E_{7(7)}$	$\mathrm{SU}(8)$	$\mathrm{SU}(4)$	$\mathrm{SU}(4)\times\mathrm{U}(1)$

- $D = 4, 5, 6a, 7$, $\mathrm{G}_{\mathrm{half}} = \mathrm{Spin}(d-1)$.

Generalised $\mathrm{G}_{\mathrm{half}}$ structure

- Define generalised $\mathrm{G}_{\mathrm{half}}$ structure in terms of tensors.
- e.g. $\mathrm{SU}(2)$ structure in 4d differential geometry

$$J_u \in \Gamma\left(\Lambda^2 T^* M\right) , \qquad J_u \wedge J_v = \delta_{uv} vol_4 , \qquad u, v = 1, \ldots, 3 .$$

Generalised $\mathrm{G}_{\mathrm{half}}$ structure

- Define generalised $\mathrm{G}_{\mathrm{half}}$ structure in terms of tensors.
- Generalisation of differential forms.
- Vector bundles $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$ whose fibres are rep spaces R_1, R_2, R_3.

D	R_1	R_2	R_3	R_c
7	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{5}$	$\emptyset$
6	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{1}$
5	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{27}$
4	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{1539}$

- For $D = 5$, $D = 6a$, $D = 7$, the $\mathrm{G}_{\mathrm{half}}$ structure takes the same form.

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- NB: $\mathrm{G}_{\mathrm{half}} \subset \mathrm{SO}(d-1,d-1) \subset E_{d(d)}$.
- "Dilaton structure": $\tilde{A} \in \Gamma\left(\mathcal{R}_2\right)$, $A \in \Gamma\left(\mathcal{R}_2^*\right)$ with

$$A \otimes A|_{\mathcal{R}_c^*} = \tilde{A} \otimes \tilde{A}|_{\mathcal{R}_c} = 0$$
$$\tilde{A}(A) = 1\,.$$

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- NB: $\mathrm{G_{half}} \subset \mathrm{SO}(d-1,d-1) \subset E_{d(d)}$.
- "Dilaton structure": $(E_{6(6)})$ A^M, $\tilde{A}_M$ with

$$A \otimes A|_{\mathcal{R}_c} = \tilde{A} \otimes \tilde{A}|_{\mathcal{R}_c^*} = 0\,, \qquad d_{MNP}A^M A^N = 0 = d^{MNP}\tilde{A}_M\tilde{A}_N\,,$$
$$\tilde{A}(A) = 1\,, \qquad \tilde{A}_M A^M = 1\,.$$

Generalised $\mathrm{G}_{\mathrm{half}}$ structure

- $\mathrm{G}_{\mathrm{half}} \subset \mathrm{SO}(d-1, d-1) \subset E_{d(d)}$.
- $\mathrm{G}_{\mathrm{half}}$ structure: $B_u{}^M \in \Gamma\left(\mathcal{R}_1\right)$.
- $u = 1, \ldots, d-1$ labels vector rep of $\mathrm{SO}(d-1)_R$.
- Compatibility conditions:

$$B_u \otimes \tilde{A}|_{\mathcal{R}_3} = 0\,,$$
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$$
\begin{aligned}
B_u \otimes \tilde{A}|_{\mathcal{R}_3} &= 0\,, \qquad & (t_\alpha)_M{}^N B_u{}^M \tilde{A}_N &= 0\,, \\
B_u \otimes B_v|_{\mathcal{R}_2} &= \tilde{A}\,\delta_{uv}\,, \qquad & B_u{}^M B_v{}^N d_{MNP} &= \tilde{A}_P\,\delta_{uv}\,,
\end{aligned}
$$

$\alpha = 1, \ldots, 78$ $E_{6(6)}$ adjoint index.

Generalised $\mathrm{G}_{\mathrm{half}}$ structure

- $\mathrm{G}_{\mathrm{half}} \subset \mathrm{SO}(d-1, d-1) \subset E_{d(d)}$.
- $\mathrm{G}_{\mathrm{half}}$ structure: ${B_u}^M \in \Gamma\left(\mathcal{R}_1\right)$.
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$$B_u \otimes \tilde{A}|_{\mathcal{R}_3} = 0\,, \qquad (t_\alpha)_M{}^N {B_u}^M \tilde{A}_N = 0\,,$$
$$B_u \otimes B_v|_{\mathcal{R}_2} = \tilde{A}\,\delta_{uv}\,, \qquad {B_u}^M {B_v}^N d_{MNP} = \tilde{A}_P\,\delta_{uv}\,,$$

- B_u, A and $\tilde{A}$ can be expressed in terms of the well-defined spinors.
- B_u define positive $(d-1)$-dimensional hyperplane in $2 \times (d-1)$-dimensional space of split signature.
- $\Rightarrow B_u$, A and $\tilde{A}$ together break $E_{d(d)} \longrightarrow \mathrm{G}_{\mathrm{half}}$
  $\Rightarrow$ define $\mathrm{G}_{\mathrm{half}}$ structure.

$\mathrm{G_{half}}$ connection and intrinsic torsion

- $\mathrm{G_{half}} \subset H_d \Rightarrow B_u$, A and $\tilde{A}$ *implicitly* define generalised metric.
- $\Rightarrow$ Possible to rewrite EFT in terms of B_u, A and $\tilde{A}$.
- Need a $\mathrm{G_{half}}$ connection $\tilde{\nabla}$.
- $\tilde{\nabla}$ satisfies

$$\tilde{\nabla} B_u = \tilde{\nabla} A = \tilde{\nabla} \tilde{A} = 0\,.$$

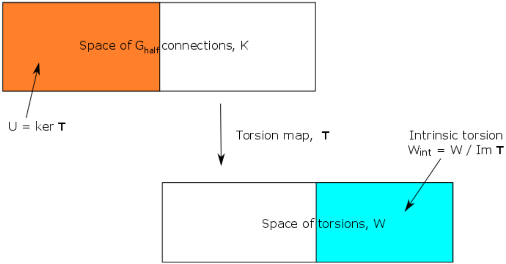
- Torsion is tensor part of the connection.

$$\mathcal{L}^{\tilde{\nabla}}_{\Lambda} V^M - \mathcal{L}_{\Lambda} V^M = \frac{1}{2} \tau_{NP}{}^M V^N \Lambda^P\,.$$

- $\tau_{MN}{}^P$ satisfies linear constraint of maximal gSUGRA.

Intrinsic $\mathrm{G}_{\mathrm{half}}$ torsion

- Torsion depends on choice of connection.



- Independent of connection: Intrinsic torsion $\in W_{int} = W/\mathrm{Im}\,T$.
- Intrinsic torsion is independent of choice of $\mathrm{G}_{\mathrm{half}}$ connection.
- Can express it without referring to a connection.

Intrinsic $\mathrm{G}_{\mathrm{half}}$ torsion

- Intrinsic torsion has "universal part":

$$S_{uv} = \mathcal{L}_{B_u} B_v \,,$$
$$T_u = \kappa^{-1} \mathcal{L}_{B_u} \kappa \,,$$

where κ is determinant of external D-dimensional metric.

- Dimension-specific part, e.g. in $D = 5$ extra singlet

$$U = \tilde{A} \left(\mathcal{L}_A A \right) \,.$$

- Intrinsic torsion because

$$\mathcal{L}_{B_u} B_v = \mathcal{L}^{\tilde{\nabla}}_{B_u} B_v - \tau \left(B_u, B_v \right) = -\tau \left(B_u, B_v \right) \,.$$

Rewriting EFT: scalar potential

- The "scalar potential" of EFT can be written as

$$V\epsilon \sim \nabla^2\epsilon\,.$$

- Use $\mathcal{N} = 2$ spinors $\epsilon^i = \Theta^i{}_{\dot\alpha}\epsilon^{\dot\alpha}$ and integrate by parts.

$$V \sim \nabla\Theta\nabla\Theta\,.$$

- Use $A, \tilde{A}, B_u \sim \Theta^2$ to rewrite

$$V \sim S_{uv}S^{uv} + T_u T^u + \ldots\,.$$

Truncation Ansatz

- So far, B_u, A, $\tilde{A}$ have arbitrary coordinate dependence.
- Truncation by expanding B_u, A, $\tilde{A}$ in terms of finite number of sections defining $\mathrm{G}_{\mathrm{half}}$ structure.
- $n(Y) \in \mathcal{R}_2^*$, $\tilde{n}(Y) \in \mathcal{R}_2$, $\omega_A(Y) \in \mathcal{R}_1$ satisfying

$$
\begin{aligned}
n \otimes n|_{\mathcal{R}_c^*} &= \tilde{n} \otimes \tilde{n}|_{\mathcal{R}_c} = 0\,, \qquad \tilde{n}(n) = 1\,, \\
\omega_A \otimes \tilde{n}|_{\mathcal{R}_3} &= 0\,, \\
\omega_A \otimes \omega_B|_{\mathcal{R}_2} &= \tilde{n}\,\eta_{AB}\,.
\end{aligned}
$$

- 3rd condition $\Rightarrow \omega_A \in V_{\mathrm{SO}(d-1,d-1)}$.
- $\Rightarrow (V_{\mathrm{G}_{\mathrm{half}}}, \mathbf{1}) \oplus (\mathbf{1}, V_{G_R}) \in \mathrm{G}_{\mathrm{half}} \times G_R$.
- $A, B = 1, \ldots, d+n-1$; $n \to$ number of vector multiplets.
- η_{AB} is $\mathrm{SO}(d-1,n)$ metric.
- ω_A maps $E_{d(d)}$ to $\mathrm{SO}(d-1,n)$ global symmetry group of gSUGRA.

Scalar truncation Ansatz

- Want $\frac{1}{2}$-max SUSY $\Rightarrow$ no $S_{\mathrm{G_{half}}}$ reps!
- Scalar truncation Ansatz:

$$A(x,Y) = e^{-2d(x)/3}\, n(Y)\,,$$
$$\tilde{A}(x,Y) = e^{2d(x)/3}\, \tilde{n}(Y)\,,$$
$$B_u(x,Y) = e^{d(x)/3}\, b_u{}^A(x)\, \omega_A(Y)\,.$$

- Compatibility conditions $\Rightarrow b_u{}^A b_{v,A} = \delta_{uv}$.
- $d(x) \to$ dilaton of $1/2$-max gSUGRA.
- Identify $b_u{}^A(x)$ related by R-symmetry.
- $P_-^{AB} = b_u{}^A b^{u,B} = \frac{1}{2}\left(\eta^{AB} - \mathcal{H}^{AB}\right) \to \mathrm{SO}(d-1,n)$ generalised metric of $1/2$-max gSUGRA.

Gauge field truncation Ansatz

- In all dimensions, obtain $d-1+n$ vector fields:

$$A_\mu{}^M(x,Y) = \mathcal{A}_\mu{}^A(x)\,\rho(Y)\,\omega_A{}^M(Y)\,.$$

- Other gauge fields depend on number of dimensions.
- E.g. $D=5$

$$A_\mu^{0\,M}(x,Y) = \mathcal{A}_\mu^0(x)\,\rho(Y)\,\tilde{n}^M(Y)\,,$$
$$B_{\mu\nu,M}(x,Y) = \mathcal{B}_{\mu\nu}(x)\,\rho(Y)^2\,n_M(Y)\,,$$

Embedding tensor and consistency

- Embedding tensor is intrinsic torsion of background $X_{ABC}(\omega, n, \tilde{n})$.
- NB: 1/2-max $\Leftrightarrow$ no $S_{\mathrm{G_{half}}}$ reps.
- For consistency, all $S_{\mathrm{G_{half}}}$ reps of intrinsic torsion must vanish, e.g.

$$\mathcal{L}_{\omega_A}\omega_B \otimes \tilde{n}\big|_{\mathcal{R}_3} = 0\,.$$

- Closure conditions, e.g.

$$\mathcal{L}_{\omega_A}\omega_B{}^M = \left(\mathcal{L}_{\omega_A}\omega_B{}^N\right)\tilde{\omega}^C{}_N\omega_C{}^M\,,$$
$$\mathcal{L}_{\omega_A} n = \tilde{n}\left(\mathcal{L}_{\omega_A} n\right) n\,.$$

- We defined the sections $\tilde{\omega}_{A,M} = (\omega_A \otimes n)_{\mathcal{R}_1^*}$.

Embedding tensor and consistency

- Remaining combinations give embedding tensor, *must be constant.*
- Details vary by dimensions, but universal contributions:

$$f_{ABC} = \mathcal{L}_{\omega_{[A}} \tilde{\omega}_{B|M|} \omega_{C]}{}^{M} \,,$$
$$f_A = \tilde{n} \left(\mathcal{L}_{\omega_A} n \right) \,,$$
$$\xi_A = \rho^{-1} \mathcal{L}_{\omega_A} \rho \quad (= 0 \text{ here}) \,.$$

- X_{ABC} satisfies linear constraint of 1/2-max gSUGRA, e.g.

$$\mathcal{L}_{\omega_{(A}} \tilde{\omega}_{B)M} \omega_C{}^M \sim \eta_{AB} f_C + f_{(A} \eta_{B)C} \,.$$

$D = 5$ example

- Example: $D = 5$ embedding tensor. Have n^M, $\tilde{n}_M$, ${\omega_A}^M$ and define $\tilde{\omega}_{A,M} = d_{MNP} n^N {\omega_A}^P$.
- Write

$$ {\omega_{\mathcal{A}}}^M = \left(n^M, {\omega_A}^M \right) , \qquad \tilde{\omega}_{\mathcal{A},M} = \left(\tilde{n}_M, \tilde{\omega}_{A,M} \right) , \quad \mathcal{A} = 0, \ldots, 5 + n . $$

- Define

$$ X_{\mathcal{ABC}} = \mathcal{L}_{\omega_{\mathcal{A}}} {\omega_{\mathcal{B}}}^M \tilde{\omega}_{\mathcal{C},M} . $$

- Only non-zero elements are

$$ X_{A00} = f_A , \quad X_{0AB} = -f_{AB} , \quad X_{ABC} = -f_{ABC} - \frac{1}{2} \eta_{AB} f_C + \eta_{C(A} f_{B)} , $$

  with $f_{AB} = f_{[AB]}$ and $f_{ABC} = f_{[ABC]}$.
- This is exactly the 5-D $\mathrm{SO}(5, n)$ embedding tensor.

Consistent truncation

- Applying truncation to EFT action we obtain 1/2-max gauged SUGRA action.
- E.g. EFT scalar potential $V \sim S_{uv} S^{uv} + T_u T^u + \ldots$.

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- E.g. EFT scalar potential $V \sim S_{uv}S^{uv} + T_u T^u + \ldots$.
- E.g. D=5 scalar potential

$$
\begin{aligned}
|e|^{-1}V = e^{-2d} f_{ABC} f_{DEF} & \left(-\frac{1}{12}\mathcal{H}^{AD}\mathcal{H}^{BE}\mathcal{H}^{CF} + \frac{1}{4}\eta^{AD}\eta^{BE}\mathcal{H}^{CF} \right. \\
& \left. -\frac{1}{6}\eta^{AD}\eta^{BE}\eta^{CF} \right) - \frac{1}{8} e^{-2d}\mathcal{H}^{AB} f_A f_B \\
& + \frac{1}{4} e^{4d} f_{AB} f_{CD} \left(\eta^{AC}\eta^{BD} - \mathcal{H}^{AC}\mathcal{H}^{BD} \right) \\
& - \frac{\sqrt{2}}{3} e^{d} f_{ABC} f_{DE} \mathcal{H}^{ABCDE} \,,
\end{aligned}
$$

where $\mathcal{H}^{ABCDE} = \epsilon^{uvwxy} b_u{}^A b_v{}^B b_w{}^C b_x{}^D b_y{}^E$.

Consistent truncation – summary

- Truncation is consistent given closure condition, spinor condition and constant embedding tensor.
- Embedding tensor automatically satisfies linear constraint of 1/2-max gSUGRA.
- Section condition $\Rightarrow$ quadratic constraint.
- With truncation Ansatz, action reduces to that of 1/2-maximal gSUGRA.
- Examples:
  - ungauged 7-d $\mathrm{SO}(3,19)$ SUGRA from M-theory on $K3$
  - ungauged non-chiral 6-d $\mathrm{SO}(4,20)$ SUGRA from M-theory on $K3 \times S^1$.
  - chiral 6-d SUGRA from IIB on $K3$.

From truncation to heteretic DFT

- Consider using truncation Ansatz but still keeping dependence on Y.
- c.f. massive IIA from EFT.

$$
\begin{aligned}
A(x,Y) &= e^{-2d(x,Y)/3}\, n(Y)\,,\\
\tilde{A}(x,Y) &= e^{2d(x,Y)/3}\, \tilde{n}(Y)\,,\\
B_u(x,Y) &= e^{d(x,Y)/3}\, b_u{}^A(x,Y)\, \omega_A(Y)\,.
\end{aligned}
$$

- But restrict ∂_M derivatives by

$$
(n\otimes\partial)\,|_{\mathcal{R}_3^*} d = (n\otimes\partial)\,|_{\mathcal{R}_3^*} b_u{}^A = 0\,.
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- E.g. $D = 5$

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$$n^M \left(t_\alpha\right)_M{}^N \partial_N d = n^M \left(t_\alpha\right)_M{}^N \partial_N b_u{}^A = 0\,.$$

- Assume derivatives can be expanded in $\tilde{\omega}_A$.

$$\partial_M d = \tilde{\omega}^A{}_M \omega_A{}^N \partial_N d\,, \qquad \partial_M b_u{}^A = \tilde{\omega}^B{}_M \omega_B{}^N \partial_N b_u{}^A\,.$$

- Theory has only 16 supercharges.

Heterotic "doubled space"

- Define $d - 1 + n$ twisted derivatives

$$D_A = \omega_A{}^M \partial_M \,, \qquad [D_A, D_B] = f_{AB}{}^C D_C + \text{ section condition } .$$

- Impose $f_{AB}{}^C D_C = 0$ and section condition.
- Can write $D_A \longrightarrow \partial_A$ (d+n-1 derivatives of heterotic DFT).
- $f_{AB}{}^C \partial_C = 0$ is required in Heterotic DFT.

Heterotic generalised Lie derivative

- Consider $V^M(x,Y) = V^A(x,Y)\,\omega_A{}^M(Y)$, $\Lambda^M(x,Y) = \Lambda^A(x,Y)\,\omega_A{}^M(Y)$.
- Generalised Lie derivative becomes

$$\mathcal{L}_\Lambda V^M = \omega_A{}^M \Big(\Lambda^B \partial_B V^A - V^B \partial_B \Lambda^A + \eta^{AB} \eta_{CD} V^C \partial_B \Lambda^D + f^A{}_{BC} V^B \Lambda^C \Big)$$

- Heterotic generalised Lie derivative & f_{ABC} encodes gauge group!
- Section condition becomes $\eta^{AB} \partial_A \otimes \partial_B = 0$.
- Restricts dependence to $d-1$ coordinates.
- Action becomes Heterotic DFT action.
- $b_u{}^A \longrightarrow$ "frame-like formulation of DFT".

M-theory / Heterotic duality

- Consider $\mathrm{SL}(5)$ EFT on $K3 \times T^6$.
- $K3 \Rightarrow 22\ \omega_A{}^M$ with $f_{ABC} = f_A = 0$.
- $T^6 \Rightarrow b_u{}^A(x, Y)$ and $d(x, Y)$ are independent of Y.
- Two ways to approach this.
- 1: Remove $T6$ by section condition $\Rightarrow \mathrm{SL}(5)$ EGG $\Rightarrow K3$ truncation of 11-d SUGRA.
- 2: "Remove $K3$" by section condition $\Rightarrow \mathrm{SO}(3, 19)$ Heterotic DFT with $\mathrm{U}(1)^{16}$ gauge group $\Rightarrow T^3$ truncation of Heterotic SUGRA.

Conclusions

- Half-maximal gSUGRAs can be obtained by consistent truncation on $\mathrm{G}_{\mathrm{half}}$-structure manifolds.
- Can couple to n vector multiplets $\Rightarrow \mathrm{SO}(d-1, n)$ global symmetry.
- Intrinsic torsion of background $\rightarrow$ embedding tensor.
- Automatically satisfies linear constraint.
- Section condition $\Rightarrow$ quadratic constraint.
- EFT can be reduced to heterotic DFT using $\mathrm{G}_{\mathrm{half}}$-structure manifolds.
- Matches expectations from M-theory / heterotic duality.

Outlook

- Examples!
- Uplift 7-D de Sitter vacuum.
- Less SUSY.
- Solutions in M-theory $\leftrightarrow$ heterotic, e.g. M5 on $K3 \leftrightarrow$ heterotic string.
- Better understanding of doubled structure $\leftrightarrow$ gauge enhancement?