String Theory and Double Field Theory

Why small circles and big circles are the same

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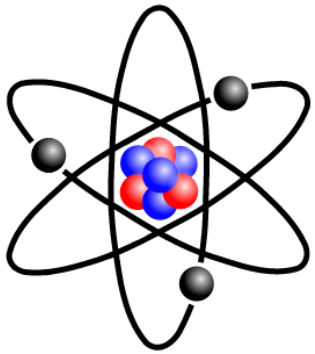
Why string theory?

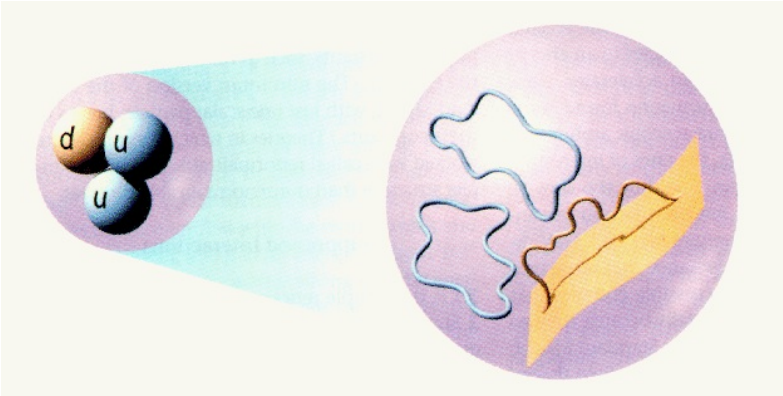
- It is currently our only proposed "theory of everything":
- Consistent with descriptions of the very large (gravity)

- and the very small (quantum mechanics)



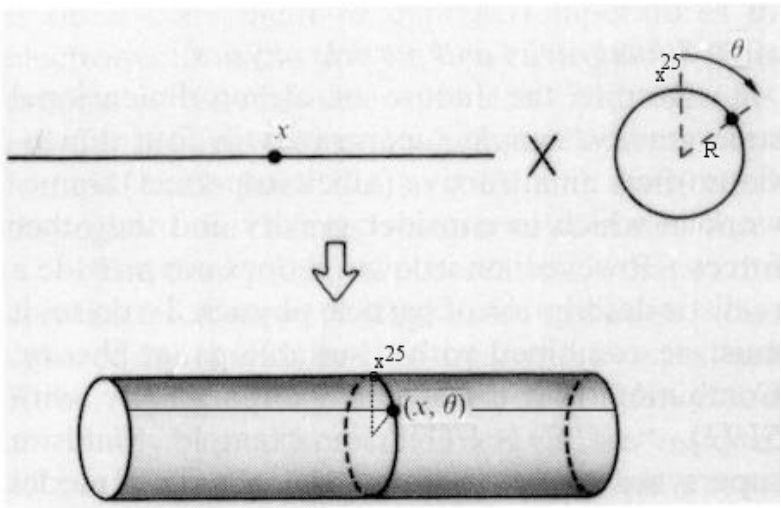
What is string theory?

- Fundamental constituents of the universe are string.
- Strings can vibrate: different modes of vibration $\rightarrow$ different particles.
- Strings have a finite extension $\rightarrow$ universe has a fundamental length scale. We cannot probe scales smaller than that.
- This fixes many difficulties of quantum mechanics. We obtain a quantum theory of gravity (so far our only quantum theory of gravity).



Why circles?

- Mathematical consistency requires strings in 10 spacetime dimensions.
- We live in four dimensions: 3 spatial + 1 time.
- Old idea: make other six dimensions into small circles.



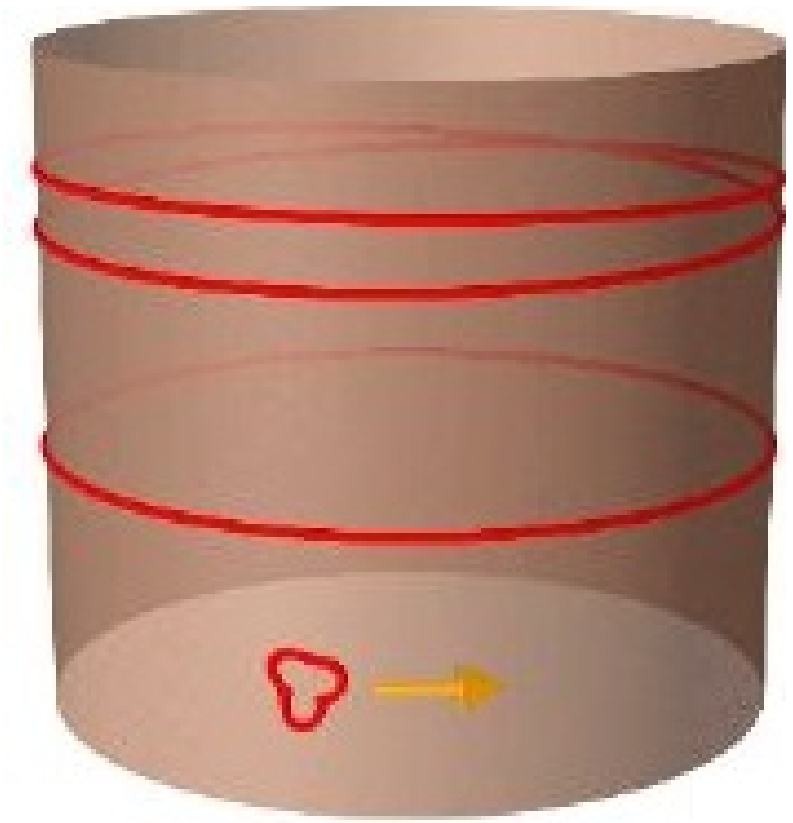
$$\mathbf{10 = 6 + 4}$$

6-d circles + 4-d spacetime. What happens in 6-d circles affects 4-d spacetime. I will talk about the 6-d circles in this talk.

Vibration, movement and winding

What can a string do when it lives on a circle?

- It can vibrate just as before $\rightarrow$ gives different particles.
- It can move along circle $\rightarrow$ this gives particles their momentum.
- It can **wind** the circle $\rightarrow$ new "degree of freedom".



Small and big

- Strings have tension so winding costs energy.
- The bigger the circle, the more stretched the string is and the more energy it costs.
- Quantum mechanics makes strings "restless": Uncertainty principle tells us the more we know where something is, the quicker it moves.
- String on a circle means we know where it is. The smaller the circle, the better we know where the string is and the faster the string will move.
- $\Rightarrow$ Small circle means a lot of momentum.

Small and big

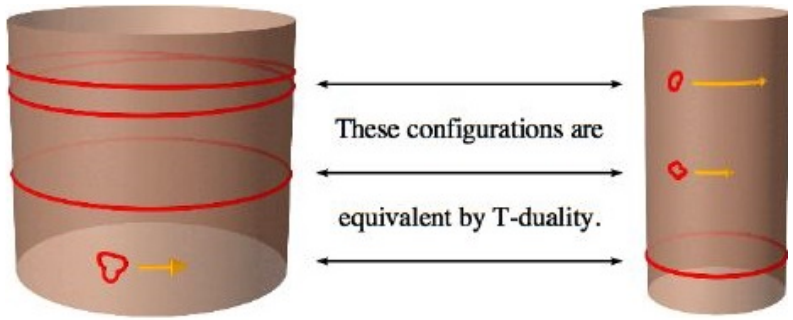
Big circle

- (Slow strings) Momentum around the circle costs little energy.
- (Big stretch) Winding the circle costs a lot of energy.

Small circle

- (Fast strings) Momentum around the circle costs a lot of energy.
- (Little stretch) Winding the circle costs little energy.

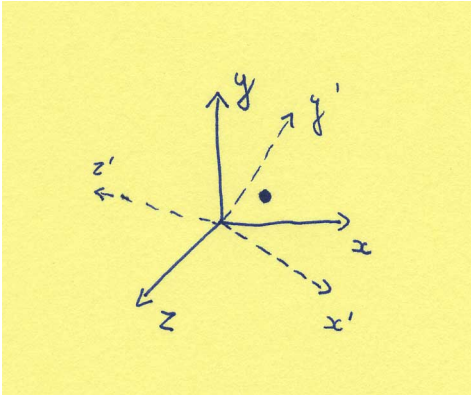
For strings big circles and small circles are the same with winding and momentum exchanged.



This is known as "T-duality".

A geometry for the string

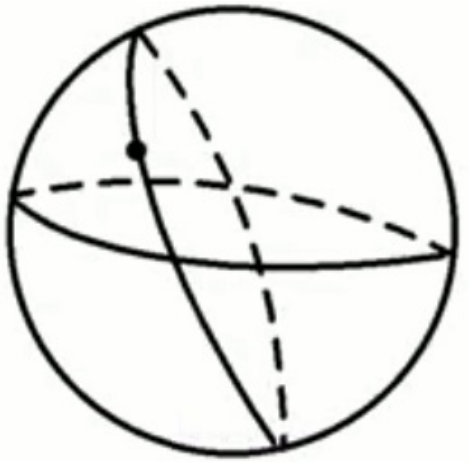
- A duality relates two mathematically different descriptions of the same situation.
- An analogy: consider a point with coordinates (x, y, z). We could rotate and instead label the point by (x', y', z').



- The rotation clearly relates two equivalent descriptions of the same physics.
- Can we do the same for the string?
- Can we write physics in a way so we see it like the strings sees it?

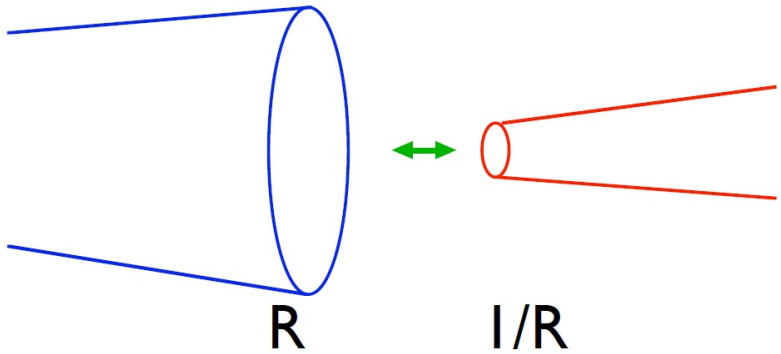
Double field theory: Physics as seen from a string

- To "geometrise" T-duality we introduce new coordinates, related to winding.
- T-dual descriptions are now related by "generalised rotations".
- We impose a condition that we always have to live on a 6-dimensional section. T-duality rotates different sections into each other.
- Recall: $\mathbf{10} = \mathbf{6} + \mathbf{4}$ and I am talking about the 6-d circles.



Why should we care?

- $10 = 6 + 4$
- What happens in the 6 "internal" dimensions affects the 4-d spacetime you get.
- String theory sees no difference between small and big circles.
- For a compact dimension we could imagine gluing a small circle to a big circle and exchange momentum with winding: T-folds.



- We need to understand geometry of T-duality to study the T-folds.
- In general, T-folds need winding cooordinates: not just a mathematical trick.

Why should we care? Open questions

- "Stringy" solutions? Predictions?
- $\mathbf{10 = 6 + 4}$
- Our universe has "cosmological constant" that is positive. String theory does not have a cosmological constant.
- $\mathbf{0} = (-\Lambda) + \Lambda$: if we can turn on negative cosmological constant in 6-d circles, we could get our universe.
- This procedure does not work without T-folds.
- String theory = T-fold + our universe ?

Why should we care? Big and small, weak and strong without strings

- So far, everything has been based around the existence of strings.
- In fact, many theories, including Einstein's General Relativity, are contained within string theory.
- These theories inherit part of the duality invariance.
- Einstein's General Relativity does not care about small and big circles either!

The end