Double field theory and non-geometry

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Outline

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Bibliography

DFT based on a variety of fields:

- Phys.Lett. B242 (1990), Nucl.Phys. B350 (1991), Nucl.Phys. B335 (1990), hep-th/93002036, hep-th/9305073 (original ideas of A. Tseytlin, M. Duff and W. Siegel)
- arXiv:1006.4823 (basis of many recent DFT papers)
- arXiv:1305.1907, arXiv:1306.2643 (reviews of DFT)
- N. Hitchin, M. Gualtieri, D. Waldram et al for generalised geometry

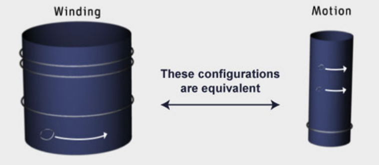
Outline

A brief review of T-duality

String theory is a theory of extended objects.
String theorists like compact spaces.
Strings on compact spaces

- can have momentum along compact directions
- can wind compact directions (unlike point particles)

String theory is invariant under exchange of momentum and winding.



This is known as T-duality.

T-duality changes the radius of the compact direction, e.g. small circle and big circle (and swaps IIA and IIB).

10-D SUGRA background transforms according to Buscher rules.

Simplest example: massless NS-NS fields (common bosonic sector of type IIA and IIB), $g_{(\mu\nu)}, B_{[\mu\nu]}, \phi$.
Buscher rules for T-duality along y are

$$\begin{aligned} g_{yy} &\to \frac{1}{g_{yy}}\,, & B_{yi} &\to -\frac{g_{yi}}{g_{yy}}\,, \\ g_{iy} &\to \frac{B_{yi}}{g_{yy}}\,, & B_{ij} &\to B_{ij} - \frac{g_{yi}B_{yj} - g_{yj}B_{yi}}{g_{yy}}\,, \\ g_{ij} &\to g_{ij} - \frac{g_{iy}g_{jy} - B_{iy}B_{jy}}{g_{yy}}\,, & \phi &\to \phi - \frac{1}{2}\ln\left|g_{yy}\right|\,. \end{aligned} \tag{1}$$

T-duality: Key points

- String spectrum & partition function remains invariant under T-duality.
- Compactified string (SUGRA) solutions are mapped into each other under T-duality.
- T-dualities form a symmetry of SUGRA actions which is hidden in standard formulation.
- These symmetries form the group $\mathrm{O(D,D)}$ for D commuting Killing vectors.
- T-duality is inherently stringy: arises from extended nature of string.

Some questions

Why do these groups appear and do they have a fundamental role in string theory (M-theory)?

What is the role of isometries?

What can we learn about string theory / SUGRA?

Can we get viable phenomenological string models?
e.g. deSitter / inflation, moduli stabilisation

Outline

A brief look at non-geometry

Toy model.

a) Consider flat T^3 with $H = dB$ flux:

$$ds^2 = dx^2 + dy^2 + dz^2 \,, \qquad B_{yz} = Nx \,. \tag{2}$$

b) T-duality along y gives twisted torus :

$$ds^2 = dx^2 + dy^2 + (dz + Nxdy)^2 \,. \qquad B_{yz} = 0 \,. \tag{3}$$

As $x \to x + 2\pi$, $z \to z - 2\pi Ny$.

c) A further T-duality along z gives non-geometry:

$$ds^2 = \frac{1}{1 + N^2x^2} \left(dz^2 + dy^2\right) + dx^2 \,, \qquad B_{yz} = \frac{Nx}{1 + N^2x^2} \,. \tag{4}$$

As $x \to x + 2\pi$, metric and B-field are *not* periodic, even up to $\mathrm{SL}(2)$ transformation.

T-folds: patching with T-dualities

We can understand what is happening as follows. View T^3 as $S^1 \times T^2$

1.) As we go around the S^1, $x \to x + 2\pi$, we patch the B-field with a gauge transformation: $B_{yz} \to B_{yz} - 2\pi N$.

2.) As we go around the S^1, we patch with a $SL(2)$ transformation $z \to z - 2\pi N y$.

3.) As we go around the S^1, we patch with a T-duality transformation.

T-duality acts on the patching: label patching transformation by Δ. Then after T-duality, represented by O, $\Delta \to O^{-1} \Delta O$.

Such backgrounds are called T-folds. Duality is a symmetry of string theory $\Rightarrow$ acceptable patching.

What is the low-energy dynamics of these backgrounds?

Outline

Enter double field theory

T-duality mixes g and B and momentum and winding.

To make T-duality manifest, we need to combine g and B and promote winding to same role as momentum.

Add D coordinates $\tilde{x}_\mu$ to represent winding: $X^A = \begin{pmatrix} x^\mu \\ \tilde{x}_\mu \end{pmatrix}$. "String moving on large and small circle."

Combine metric and Kalb-Ramond form into an $\mathrm{O(D,D)}$ element so it transforms as a tensor under $\mathrm{O(D,D)}$: generalised metric

$$M_{AB} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\kappa} B_{\kappa\nu} & -B_{\mu\rho} g^{\rho\nu} \\ g^{\mu\rho} B_{\rho\nu} & g^{\mu\nu} \end{pmatrix} . \tag{5}$$

$\mu, \nu = 1, \ldots D$ are spacetime indices.
$A, B = 1, \ldots 2D$ are $\mathrm{O(D,D)}$ indices.

Enter double field theory

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$$M_{AB} = \begin{pmatrix} M_{\mu\nu} & M_\mu{}^\nu \\ M^\mu{}_\nu & M^{\mu\nu} \end{pmatrix} . \tag{6}$$

$\mu, \nu = 1, \dots D$ are spacetime indices.
$A, B = 1, \dots 2D$ are $\mathrm{O(D,D)}$ indices.

Conservatives and optimists

$$S = S_{10-D} + S_D \,. \tag{7}$$

Conservatively: S_{10-D} is our low-energy dynamics ($D = 6$). S_D is compact and described by DFT. It describes the scalar potential of the compactification.

Optimistic: We will see that allowing extra coordinates means we don't need isometries / compactifications. We can make $D = 10$ and describe all directions by DFT.

We should understand the situation $\tilde{\partial}^\mu = 0$. Then generalised tensors live in $TM \oplus T^*M$ and this is the realm of "generalised geometry". (Hitchin, Gualtieri, ..., Waldram, ...)

Different ways to see this is needed:

- $TM \oplus T^*M$ has a natural action of $\mathrm{O}(\mathrm{D},\mathrm{D})$.
  For $v, w \in TM$, $\xi, \rho \in T^*M$ $\langle v + \xi | w + \rho \rangle = \xi(w) + \rho(v)$.
  The $\mathrm{O}(\mathrm{D},\mathrm{D})$ metric $\langle \, | \, \rangle$ is denoted by

$$\eta_{AB} = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} . \qquad (8)$$

- Metric + 2-form have diffeo + gauge symmetry.
  Diffeomorphisms are infinitesimally generated by vectors.
  Gauge transformations are infinitesimally generated by one-forms.

DFT is an "$\mathrm{O}(\mathrm{D},\mathrm{D})$-covariantisation" of this by including winding coordinates $\tilde{x}_\mu$. This will give us an action.

Generalised metric satisfies

$$M_{AC}\eta^{CD}M_{DB}=\eta_{AB}\,, \tag{9}$$

and is unit determinant.

We need another object to have a volume element. This is the generalised dilaton d, an $\mathrm{O(D,D)}$ scalar

$$e^{-2d}=\sqrt{g}e^{-2\phi}\,, \tag{10}$$

where ϕ is string theory dilaton.

Generalised diffeomorphisms

Recall: diffeomorphisms and gauge transformations are generated by a vector ξ^μ + covector λ_μ = generalised vector $U^A = (\xi^\mu, \lambda_\mu)$.
Collective symmetries described by generalised Lie derivative

$$\mathcal{L}_U V^A = U^B \partial_B V^A - V^B \partial_B U^A + \eta^{AB} V^C \eta_{CD} \partial_B U^D \, . \tag{11}$$

For generalised metric, this gives exactly diffeo + gauge symmetry when you set $\tilde{\partial}^\mu = \frac{\partial}{\partial \tilde{x}_\mu} = 0$.

In GR, the algebra of diffeomorphisms closes:

$$\left[L_U, L_V \right] W^A = L_{[U,V]} W^A \tag{12}$$

where [,] is the commutator and Lie bracket, and L the Lie derivative.

Section condition

Does the algebra of generalised diffeomorphisms close?

NO!

$$[\mathcal{L}_U, \mathcal{L}_V]\, W^A = \mathcal{L}_{[U,V]_C} W^A + \mathrm{junk} \tag{13}$$

YES! if one imposes the "section condition"

$$\eta^{AB} \partial_A f \partial_B g = 0\,. \tag{15}$$

for any two fields f, g in the theory. This is also "level-matching" condition of string theory.

This condition restricts dependence of any field $f(x^\mu, \tilde{x}_\mu)$ so that f depends at most on D of the coordinates and never on x^z and its dual $\tilde{x}_z$ at the same time (z is a label for a specific index).

This is an $\mathrm{O(D,D)}$ covariantisation of the requirement $\tilde{\partial}^\mu = 0$.
Both $\tilde{\partial}^\mu = 0$ and $\partial_\mu = 0$ are solutions.

Section condition

Does the algebra of generalised diffeomorphisms close?

Weakly

$$[\mathcal{L}_U, \mathcal{L}_V]\, W^A \approx \mathcal{L}_{[U,V]_c} W^A \tag{14}$$

YES! if one imposes the "section condition"

$$\eta^{AB} \partial_A f \partial_B g \approx 0 \,. \tag{16}$$

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Outline

The action

We can write a unique action in terms of $\mathrm{O(D,D)}$ tensors that is weakly a scalar under generalised Lie derivative.

$$S = \int dx d\tilde{x}\; e^{-2d} \left(\frac{1}{8} M^{AB} \partial_A M^{CD} \partial_B M_{CD} - \frac{1}{2} M^{AB} \partial_A M^{CD} \partial_C M_{BD} \right.$$
$$\left. + 4 M^{AB} \partial_A \partial_B d - \partial_A \partial_B M^{AB} - 4 M^{AB} \partial_A d \partial_B d + 4 \partial_A M^{AB} \partial_B d \right) . \qquad (17)$$

$$\mathcal{L}_U S \approx 0 \, . \qquad (19)$$

The action

We can write a unique action in terms of $\mathrm{O(D,D)}$ tensors that is weakly a scalar under generalised Lie derivative.

$$S = \int dx d\tilde{x} \; e^{-2d} \left(\partial M\right)^2 \,. \tag{18}$$

$$\mathcal{L}_U S \approx 0 \,. \tag{19}$$

A bit of magic

If we now consider the solution of the section condition $\tilde{\partial}^\mu = 0$ we get

$$S = \int dx \; e^{-2\phi} \sqrt{g} \left(R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - 4 \nabla_\mu \phi \nabla^\mu \phi + 4 \Box \phi \right) , \qquad (20)$$

where $H = dB$ and R is the Ricci scalar. All indices raised / lowered with $g_{\mu\nu}$. This is the action for the NS-NS sector we are considering.

Outline

Non-geometry and fluxes

We recall the toy model $T^3 = S^1 \times T^2$:

H-flux $\rightarrow$ twisted torus ("geometric flux") $\rightarrow$ "non-geometric flux".

Each arrow is a T-duality.

We said that T-duality transforms the patching. Explicitly in DFT:

$$M_{AB}(x + 2\pi) \rightarrow \Delta^T M_{AB}(x)\Delta \,, \tag{21}$$

where $\Delta \in \mathrm{O}(2,2)$ corresponds to the different patchings (B-field, $SL(2)$, T-duality).

1. The H-flux:

$$(\Delta_1)^A{}_B = \begin{pmatrix} \delta^\mu{}_\nu & 0 \\ \delta B_{\mu\nu} & \delta_\mu{}^\nu \end{pmatrix} , \qquad (22)$$

where $\delta B_{xy} = -2\pi N$ is the gauge transformation patching.

2. The "geometric flux":

$$(\Delta_2)^A{}_B = \begin{pmatrix} A^\mu{}_\nu & 0 \\ 0 & \left(A^{-T}\right)_\mu{}^\nu \end{pmatrix} , \qquad (23)$$

where $A_\mu{}^\nu$ is the $SL(2)$ transformation patching.

3. The "non-geometric flux":

$$(\Delta_3)^A{}_B = \begin{pmatrix} \delta^\mu{}_\nu & \delta\beta^{\mu\nu} \\ 0 & \delta_\mu{}^\nu \end{pmatrix} , \qquad (24)$$

where $\beta^{yz} = -2\pi N$.

O_{12} is the generator of the T-duality going between 1 and 2, $\Delta_2 = (O_{12})^{-1}\,\Delta_1\ O_{12}$, etc.

Choice of frame

The first two patchings, Δ_1, Δ_2, look natural in terms of $g_{\mu\nu}, B_{\mu\nu}$: they are just diffeos + gauge transformations.

Since T-duality is a symmetry of strings, how can we see Δ_3 as being natural?

Define generalised vielbeine:

$$M_{AB} = E_A{}^i E_B{}^j \delta_{ij} \tag{25}$$

Because of $M\eta^{-1}M = \eta$, the vielbeine satisfy

$$\eta_{AB} = E_A{}^i E_B{}^j \eta_{ij} \,. \tag{26}$$

Therefore, local $\mathrm{O(D)} \times \mathrm{O(D)}$ can act on the i, j flat indices and preserve these defining equations.

Choice of frame

The parameterisation

$$M_{AB} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} B^{\rho}{}_{\nu} & B_{\mu}{}^{\nu} \\ -B^{\mu}{}_{\nu} & g^{\mu\nu} \end{pmatrix} \tag{27}$$

corresponds to the choice of vielbein

$$E_A{}^i = \begin{pmatrix} e & 0 \\ eB & e^{-T} \end{pmatrix} , \tag{28}$$

where $e^T e = g$. However, an equivalent choice, related by an $\mathrm{O(D)} \times \mathrm{O(D)}$ transformation gives

$$E_A{}^i = \begin{pmatrix} \tilde{e} & \beta \tilde{e} \\ 0 & \tilde{e}^{-T} \end{pmatrix} , \tag{29}$$

where $\beta^{\mu\nu}$ is a “bivector” and $\tilde{e}$ is in principle different from e.

We see that double field theory contains in general new fields in the "supergravity"!

These appear on the same footing as the usual $g_{\mu\nu}$, $B_{\mu\nu}$ fields.

The DFT action, being written in terms of M_{AB}, describes the dynamics of these fields.

Can these new fields play a role in understanding non-geometry?

The generalised Lie derivative tells us how these fields transform under gauge transformations. These expressions look nice in the section $\partial_\mu = 0$, e.g.

$$\delta_U \beta^{\mu\nu} = U_\rho \tilde{\partial}^\rho \beta^{\mu\nu} + \beta^{\kappa\nu} \tilde{\partial}^\mu U_\kappa + \beta^{\mu\kappa} \tilde{\partial}^\nu U_\kappa + 3\tilde{\partial}^{[\mu} U^{\nu]} \,. \tag{30}$$

This is just the gauge transformation laws for $B_{\mu\nu}$ with all indices reversed.

Under

$$\Delta_3 = \begin{pmatrix} 1 & \delta\beta \\ 0 & 1 \end{pmatrix} , \tag{31}$$

we just have $\beta^{\mu\nu} \to \beta^{\mu\nu} + \delta\beta^{\mu\nu}$. Thus, this frame is a natural choice for 3:

$$d\tilde{s}^2 = dx^2 + dy^2 + dz^2 \,, \qquad \beta^{yz} = Nx \,. \tag{32}$$

We see that this is the reverse of the H-flux example (1).

Fluxes: geometric and non-geometric

What is the analogous "field strength" to H? It is

$$Q^{\nu\rho}{}_{\mu} = \partial_\mu \beta^{\nu\rho} \,. \tag{33}$$

The different fluxes are all contained in the "generalised torsion":

$$(\mathcal{L}_{E_i} E_j)^A = \tau^A{}_{BC} E^B{}_i E^C{}_j \,. \tag{34}$$

This is not $\mathrm{O(D)} \times \mathrm{O(D)}$ invariant! In the usual frame

$$E = \begin{pmatrix} e & 0 \\ eB & e^{-T} \end{pmatrix} \,, \tag{35}$$

we find the "geometric fluxes"

$$\begin{aligned} \tau^\mu{}_{\nu\rho} &\sim T^\mu{}_{\nu\rho} \,, & \tau_{\mu\nu\rho} &\sim H_{\mu\nu\rho} \,, \\ \tau_{\mu\nu}{}^\rho &= 0 \,, & \tau^{\mu\nu\rho} &= 0 \,. \end{aligned} \tag{36}$$

where $T \sim [e, e]$.

In the "reversed" frame with β

$$E = \begin{pmatrix} e & \beta e^{-T} \\ 0 & e^{-T} \end{pmatrix} , \qquad (37)$$

we have instead

$$\begin{aligned} \tau^{\mu}{}_{\nu\rho} &= 0 \,, \\ \tau_{\mu\nu\rho} &= 0 \,, \\ \tau_{\mu}{}^{\nu\rho} &\sim Q^{\nu\rho}{}_{\mu} \,, \\ \tau^{\mu\nu\rho} &\sim R^{\mu\nu\rho} \,, \end{aligned} \qquad (38)$$

where $R^{\mu\nu\rho} = \tilde{\partial}^{[\mu}\beta^{\nu\rho]}$.
When talking about fluxes we need to make sure they are globally well-defined. Consider the toy model:

In the third scenario with the T-duality patching, neither $T^{\mu}{}_{\nu\rho}$ nor $H_{\mu\nu\rho}$ are well-defined when going around $x \to x + 2\pi$. But, in the "reversed" frame, $Q^{yz}{}_{x}$ is globally well-defined. Thus, "reversed frame" is preferred!

We have found a new object $R^{\mu\nu\rho} = \tilde{\partial}^{[\mu}\beta^{\nu\rho]}$. What is it?

Recall the toy model. Calculating the fluxes in a frame so that they are globally well-defined, we find each T-duality raised an index on the flux:

$$H_{yzx} \to T^{y}{}_{zx} \to Q^{yz}{}_{x} \,. \tag{39}$$

We have run out of directions to T-dualise to get $R^{\mu\nu\rho}$. However, in DFT we can T-dualise along non-isometries and get the final step.

$$Q^{yz}{}_{x} \to R^{yzx} \,. \tag{40}$$

This solution explicitly involves the dual coordinates $\tilde{x}$.

This chain of dualities involved fully geometric backgrounds as well as non-geometric ones.

Can we find "fully non-geometric" backgrounds where T-duality does not give a geometric background?

We have "geometrical" objects $\tau^{A}{}_{BC}$ that in some frames capture the fluxes of these backgrounds.

What is the "characteristic class" that picks up the patching?

What next?

- DFT describes the low-energy dynamics of all the fluxes, including "non-geometric" Q and R.
- We can describe them geometrically.
- These non-geometric fluxes are important in providing a higher-dimensional origin for all gauged supergravities (some were previously orphaned).
- Non-geometric compactifications could give interesting scalar potentials, e.g. moduli stabilisation, deSitter, inflation?
- What sources these non-geometric fluxes? Exotic branes?
- Relax section condition and move beyond SUGRA. More than a rewriting.
- M-theory and U-duality.

Outline

Geometry

The action of DFT is a great success: it is manifestly T-duality invariant, it is weakly a generalised scalar and it reproduces the right SUGRA action in the right duality frame.

$$S \sim \int dx d\tilde{x} \, (\partial M)^2 \ . \qquad (41)$$

However, it is not manifestly geometric: the terms are not tensors under generalised diffeomorphisms.

Recall: $\mathcal{L} \neq L$. Generalised diffeomorphisms are not just "doubled" diffeomorphisms!

We need to find new tensors. Analogues of Levi-civita connection fail (and curvature objects analogous to Rimenann curvature cannot be defined in M-theory extension!)

Curvature-less but torsionful connections

However, one can use a flat connection

$$\Gamma^{A}{}_{BC} = E^{A}{}_{i}\partial_{B}E_{C}{}^{i} \,. \tag{42}$$

This has vanishing curvature but non-zero torsion: $\tau^{A}{}_{BC}$.

$$\left(\mathcal{L}_{U}^{\nabla} - \mathcal{L}_{U}^{\partial}\right) V^{a} = \tau^{A}{}_{BC}U^{B}V^{C} \,. \tag{43}$$

This is the same as the equation for $\tau^{A}{}_{BC}$ given before. This is the τ which contains all the fluxes.

We can write down a "teleparallel" action in terms of this torsion

$$S \sim \int dx d\tilde{x} \;\, (\tau)^{2} \,. \tag{44}$$

Note that neither the connection

$$\Gamma^A{}_{BC} = E^A{}_i \partial_B E_C{}^i \,, \tag{45}$$

nor the torsion $\tau^A{}_{BC}$ are $\mathrm{O(D)} \times \mathrm{O(D)}$ invariant! We construct the action by requiring this local invariance at the level of the action.

$$S \sim \int dx d\tilde{x} \; \tau^2 \tag{46}$$

is for obvious reasons also known as “flux formalism” of DFT.

Scalar potential

If we compactify D dimensions, thanks to DFT, we now have the scalar potential in terms of "geometric" and "non-geometric" fluxes.

How can we turn the fluxes on?

Tori don't work because the fluxes will "push" the tori. Need to use Scherk-Schwarz compactification.

Scherk-Schwarz

Let $X = (\mathbb{X}, \mathbb{Y})$ and consider a dimensional reduction such that

$$V^A(\mathbb{X}, \mathbb{Y}) = W^A{}_b(\mathbb{Y})\, \hat{V}^b(\mathbb{X}) \,, \qquad W^A{}_b \in \mathrm{O(D,D)} \,, \tag{47}$$

and similarly for all other tensors. This gives a gauged supergravity, with the twists W entering in the action only in the combinations of

$$f^a{}_{bc} = 3\eta^{ae}\eta_{d[e} W^A{}_b W^B{}_{c]} \partial_A \left(W^{-1}\right)^d{}_B \,. \tag{48}$$

These will be the structure constants of your gauged group.

In fact, they are related to the reduced torsion components $\tau^A{}_{BC}$:

$$\tau^A{}_{BC}(\mathbb{X}, \mathbb{Y}) = W^A{}_a \left(W^{-1}\right)_B{}^b \left(W^{-1}\right)_C{}^c \left(\hat{\tau}^a{}_{bc}(\mathbb{X}) + f^a{}_{bc}\right) \,, \tag{49}$$

where $\hat{\tau}^a{}_{bc}$ is defined analogously to $\tau^A{}_{BC}$ in terms of hatted quantities.

Gauged SUGRA reductions

If we do a Scherk-Schwarz compactification on normal 10-D type II SUGRA, we do not get all lower-dimensional gauged SUGRAs.

Some lower-dimensional GSUGRAs can be obtained by directly gauging lower-dimensional SUGRAs.

All such gauged SUGRAs can be classified by the "embedding tensor" $\theta^a{}_{bc}$: how the gauged subgroup embeds in the $\mathrm{O(D,D)}$ group.

Group theory shows that $\theta^a{}_{bc}$ and $f^a{}_{bc}$ agree. But some $f^a{}_{bc}$ components are set to zero through this reduction. Why does this happen and how can we remedy this?

IS STRING THEORY NOT THE SOURCE OF EVERYTHING???

Outline

Beyond supergravity

The section condition $\eta^{AB}\partial_A \otimes \partial_B \approx 0$ means we are really only ever dealing with D dimensions.

So far everything is but a rewriting of supergravity actions! So the same facts are still true for GSUGRAs.

We can go beyond SUGRA. $\eta^{AB}\partial_A \otimes \partial_B \approx 0$ is sufficient but not necessary for consistency of theory.

A more relaxed condition can be used.

A new look at Scherk-Schwarz compactifications

All consistency requirements (closure of algebra, invariance of action under generalised diffeomorphisms, ...) can now be satisfied by:

$$\begin{aligned} \eta^{ab}\partial_a \hat{g}\left(\mathbb{X}\right)\partial_b \hat{h}\left(\mathbb{X}\right) \approx 0\,, \\ \eta^{ab}\partial_a W \partial_b \hat{g}\left(\mathbb{X}\right) \approx 0\,, \end{aligned} \tag{50}$$

and Jacobi identity

$$f^e{}_{[ab} f^f{}_{c]d} = 0\,. \tag{51}$$

We do not impose

$$\eta^{AB}\partial_A W \partial_B W \neq 0\,. \tag{52}$$

Thus, we do not impose the *full* section condition.

Non-geometry again

These relaxed compactifications contain some dependence on winding coordinates $\tilde{x}_\mu$.

This is the final ingredient to get the remaining gauged SUGRAs!

These other gauged SUGRAs have non-geometric fluxes turned on in the scalar potential.

Thus, DFT provides a higher-dimensional origin for these previously orphaned SUGRAs.