# Introduction to String Theory

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## Exercise Sheet 14

**1** Consider the effective action of two parallel, coincident D-branes

$$S = -\left(2\pi\alpha\right)^2 T_p \int d^{p+1}\xi \,\mathrm{Tr}\left(\frac{1}{4}F_{ab}F^{ab} + \frac{1}{2}\mathcal{D}_a\phi^I\mathcal{D}^a\phi^I - \frac{1}{4}\sum_{I\neq J}\left[\phi^I,\phi^J\right]^2\right). \tag{1.1}$$

**(a)** Show that taking $\phi^i = 0$ for $i = 0,\ldots,24$, $A_a = 0$ and

$$\phi^{25} = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \tag{1.2}$$

with $\phi_1$ and $\phi_2$ constant is a solution of the effective action (1.1). Interpet the configuration (1.2) in terms of the position of the D-branes.

**(b)** Compute the spectrum of the gauge fields around the vacuum (1.2). Interpret this result.

**(c)** Compute the level 1 spectrum of the quantised strings ending on the D-branes. Compare this to the spectrum obtained from the effective action in **(b)**.

**2** Consider the spacetime $\mathbb{R}^{1,24} \times S^1$ with $S^1$ of radius $R$. The most general metric can be written as

$$ds^2 = \tilde{G}_{\mu\nu}dX^\mu dX^\nu + e^{2\sigma}\left(dX^{25} + A_\mu dX^\mu\right)^2, \tag{2.1}$$

with $\mu,\nu = 0,\ldots,24$ and $X^{25}$ the local coordinate on $S^1$, such that $X^{25} \sim X^{25} + 2\pi R$.

**(a)** Let $\tilde{G}_{\mu\nu}$, $A_\mu$, and $\sigma$ be independent of the $S^1$ coordinate $X^{25}$. How do the fields $\tilde{G}_{\mu\nu}$, $A_\mu$ and $\sigma$ transform under the 25-dimensional diffeomorphisms?

**(b)** Decompose the Kalb-Ramond 2-form $B_{(2)}$ analogously to (2.1). What fields do you get in $\mathbb{R}^{1,24}$?

**(c)** Perform a Fourier expansion of the dilaton $\Phi(X^\mu, X^{25})$ along $S^1$. Evaluate the kinetic term of the dilaton

$$S_{\mathrm{kin},\Phi} = \int d^{26}X\, \partial_{\hat{\mu}}\Phi\partial^{\hat{\mu}}\Phi\,, \tag{2.2}$$

where $\hat{\mu} = 0,\ldots,25$ and we ignore the coupling to gravity, for the Fourier modes of the dilaton. Interpret this result.

**(d)** How do the Fourier modes of $\Phi(X^\mu, X^{25})$ along $S^1$ transform under 25-dimensional diffeomorphisms? Interpret this result.